

## Homework 5

due 3/28/2012

**Problem 1** Suppose market demand is given by  $p(q) = a - bq$ , and there are two firms, each with a constant marginal costs of  $c$  and no fixed cost. The two firms choose quantity simultaneously, and then sell whatever they have produced at the prevailing market price.

- a. Determine NE quantities for both firms. Demonstrate that there is only one equilibrium in this game.
- b. Derive the market price, and the profit for each firm. Show that the total quantity produced is greater than the monopoly quantity, but less than the competitive quantity.
- c. Now suppose that firm 1 moves first. Firm 2 moves only after observing the quantity firm 1 chooses. Derive the SPE of this game.
- d. Finally, suppose there are  $J$  firms serving the market (and choosing quantity simultaneously). In the static case, determine NE quantities and profits for each of the  $J$  firms. Show that as  $J \rightarrow \infty$ , total production approaches the competitive level, while when  $J = 1$ , we get the monopoly outcome.

**Problem 2** An inventor has discovered a new method of producing a precious stone, using spring water found only in Venice, Italy and Danville, Kentucky. The process is patented and manufacturing plants are set up in both places. The product is sold only in Europe and the US. Trade laws are such that the price must be uniform within Europe and the US, but the European and American prices may differ. Transport costs are negligible, and there is no second-hand market in the stones because of the risk of forgeries. From the production and marketing data given below, determine the profit-maximizing production and sales plans. In particular, determine the output in Venice and Danville, sales in the US and Europe, quantity shipped from Europe to the US or vice versa, and prices in Europe and the US.

$$\begin{aligned} \text{Demand: US, } p &= 1500 - \frac{1}{2}Q; \text{ Europe, } p = 1000 - Q \\ \text{Average cost: Danville, } AC &= 150 + .375Q; \text{ Venice, } AC = 100 + \frac{1}{2}Q \end{aligned}$$

**Problem 3** MWG, problem 12.D.3

**Problem 4** MWG, problem 12.E.4

**Problem 5** Consider an oligopoly with  $K$  firms. Each firm can produce costlessly, and market demand is given by  $P = 1 - \sum_{i=1}^K q_i$ .

- a. Suppose  $K = 3$ , and the three firms are Cournot competitors. Solve for the Nash equilibrium profits of firms 1, 2, and 3 in this game.
- b. Suppose  $K = 3$ . Suppose that firm 1 publicly commits to a quantity. After observing firm 1's choice of quantity, firms 2 and 3 simultaneously choose quantities. Solve for the subgame perfect equilibrium profits of firms 1, 2, and 3 in this game.

**For parts c-e:** Suppose that  $K$  firms choose quantity sequentially (that is, firm 1 publicly announces a quantity. After observing  $q_1$ , firm 2 publicly announces a quantity. After observing  $q_1$  and  $q_2$ , firm 3 publicly announces a quantity, and so on).

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- c.** Suppose  $K = 3$ . Solve for each firm's profit in the subgame perfect equilibrium of this game.
- d.** Now suppose  $K$  is a large number. Solve for the SPE of the game using the method of your choosing. Calculate firm  $k$ 's SPE profits.