Homework 6

answers

Problem 1 A seller has one unit of a good which she may sell to a buyer. The seller has private information about her valuation of the good, v, which is drawn from $[0, 1]$ according to the uniform distribution. When the seller's valuation of the good is v, the buyer's valuation is kv, where $k > 1$. The buyer does not observe his valuation, however, but does have accurate knowledge of the distribution of the seller's valuation. Both players are risk-neutral.

a. Suppose that the buyer makes a take-it-or-leave-it offer to the seller. That is, the buyer offers a price at which he is willing to buy, the seller either accepts or rejects, and rejection results in no sale. Describe all subgame perfect equilibria in pure strategies. How does your analysis depend on the value of k?

If the buyer makes an offer of p, the seller will accept it iff $v \leq p$. The buyer's expected utility from an offer of p is then $E[kv|v \leq p] = \frac{kp}{2}$. If $k < 2$, this is less than p for all p, and so the buyer's optimal offer is $p = 0$ (in other words, the buyer is never willing to buy). If $k \geq 2$, the buyer's expected utility as a function of his offer p is $\frac{kp}{2} - p$ for all $p \leq 1$, which is a strictly increasing function of p, and so the buyer's optimal offer is $p = 1$.

b. Suppose now that the seller makes a take-it-or-leave-it offer. That is, the seller charges a price, the buyer either accepts or rejects, and rejection results in no sale. Describe all perfect Bayesian equilibria in pure strategies. How does your analysis depend on the value of k?

A strategy for the seller is a function $p(v)$ mapping her observed value of v into a price p. Focus first on equilibria in which offer $p(v)$ is accepted for all v. Clearly, $p(v)$ cannot be increasing, as were $p(v') < p(v'')$ for some $v' < v''$ the seller with $v = v'$ would prefer to deviate from price $p(v')$ to $p(v'')$. A similar argument says that $p(v)$ cannot be decreasing. Therefore, in any such equilibrium, $p(v) = p$ for all v. Now, under Bayesian beliefs, a buyer will accept such an offer iff $p \leq \frac{k}{2}$, meaning the optimal price for the seller is $p = \frac{k}{2}$ so long as $k \geq 2$. If $k < 2$, there is no equilibrium in which the seller's offer is always accepted.

Now, there are always equilibria in which the seller's offer $p(v)$ is rejected for all v (eg. seller sets $p(v) = \$1M$ for all v, buyer always rejects, and believes that the quality behind any price other than \$1M is 0). If $k \geq 2$, there are also equilibria in which the seller's offer is accepted for v in some nonempty subset of $[0, 1]$, though the above logic implies that this subset must be $[0, \overline{v}]$ for $\overline{v} < 1$ (example: $p(v) = \frac{k}{4}$ for $v \in [0, \frac{1}{2}]$, $p(v) = \$1M$ for $v > \frac{1}{2}$, with appropriate beliefs). This last class of equilibria does not seem very compelling. So basically there are equilibria in which the item is always sold for price $\frac{k}{2}$ ($k \geq 2$ only) and equilibria in which the item is never sold (all k).

Problem 2 MWG 13.B.1

See figure at end of answer set. The graphy of $\hat{\theta}$ is simply a 45 degree line. $E(\theta|\theta < \hat{\theta})$ and $r(\hat{\theta})$ are both presumably increasing functions, but need not have any particular shape. As described in the figure, a competitive equilibrium will be located where $E(\theta|\theta < \hat{\theta}) = r(\hat{\theta})$.

Problem 3 MWG 13.B.3

a. For any wage w, a worker accepts if $r(\theta) < w$. Since $r(\theta)$ is a decreasing function, the set of workers who accepte wage w will be $[\theta^*, \overline{\theta}]$, for some θ^*.

b. If $r(\theta) > \theta$ for all θ, then the efficient outcome is for no one to work. In fact, this is what happens in the competitive equilibrium. To see this, look at the figure at the end of this answer set. If $r(\theta) > \theta$, then we have that at $w = r(\overline{\theta})$, $E(\theta|r(\theta) < w) = \overline{\theta}$ is below the 45 degree line. From there, $(\theta|r(\theta) < w)$ is a decreasing function, and so will never cross the 45 degree line. Therefore, there is no competitive equilibrium in which any workers are hired.

c. The Pareto efficeint outcome would be that the the set of workers in $[\hat{\theta}, \overline{\theta}]$ work and those in $[\underline{\theta}, \hat{\theta}]$ do not. Were this to be an equilibrium, the wage would have to be $w = \hat{\theta} = r(\hat{\theta})$. Suppose that the firm were to offer a wage of $w = r(\hat{\theta})$. Workers in $[\hat{\theta}, \overline{\theta}]$ would accept, meaning that $E[\theta|r(\theta) < w] > w$, meaning that the firm would like to hire more workers, and so would increase the wage.

Problem 4 MWG 13.C.1

First, it is clearly an equilibrium for all workers to submit to the test, and for the firms to pay a wage of $\underline{\theta}$ to any worker not doing so (no worker has an incentive to deviate since the test is costless.

Now suppose that there is some other equilibrium in which workers in set $A \subset [\underline{\theta}, \overline{\theta}]$ do not submit to the test, where A has positive measure. Necessarily, all workers in A must be paid $w = E(\theta|\theta \in A)$. Further, there must exist some $\theta \in A$ satisfying $\theta > E(\theta|\theta \in A)$. But then this worker would earn a higher wage by taking the test, breaking the proposed equilibrium.

Problem 5 Suppose that normal workers increase a firm's revenue by \$$X$, while smart workers increase revenue by \$$A$, where $A > X$. Firms cannot tell smart workers from normal workers *ex ante*, but can observe a worker's educational level.

Any worker can acquire as much education as she wishes, but getting e years costs a normal worker $B * e$, where $B > 1$, while e years cost a smart worker only e.

a. Describe the unique separating equilibrium which satisfies the inutitive criterion.

In the intuitive criterion equilibrium, normals workers must be indifferent between getting 0 years of education and e^* years, where e^* is the number of years a smart worker gets. Therefore, we must have $X = A - Be^*$, or $e^* = \frac{A-x}{B}$.

b. As A increases, does the level of education obtained by smart workers increase or decrease in the equilibrium described in part a.? Explain intuitively why this is the case.

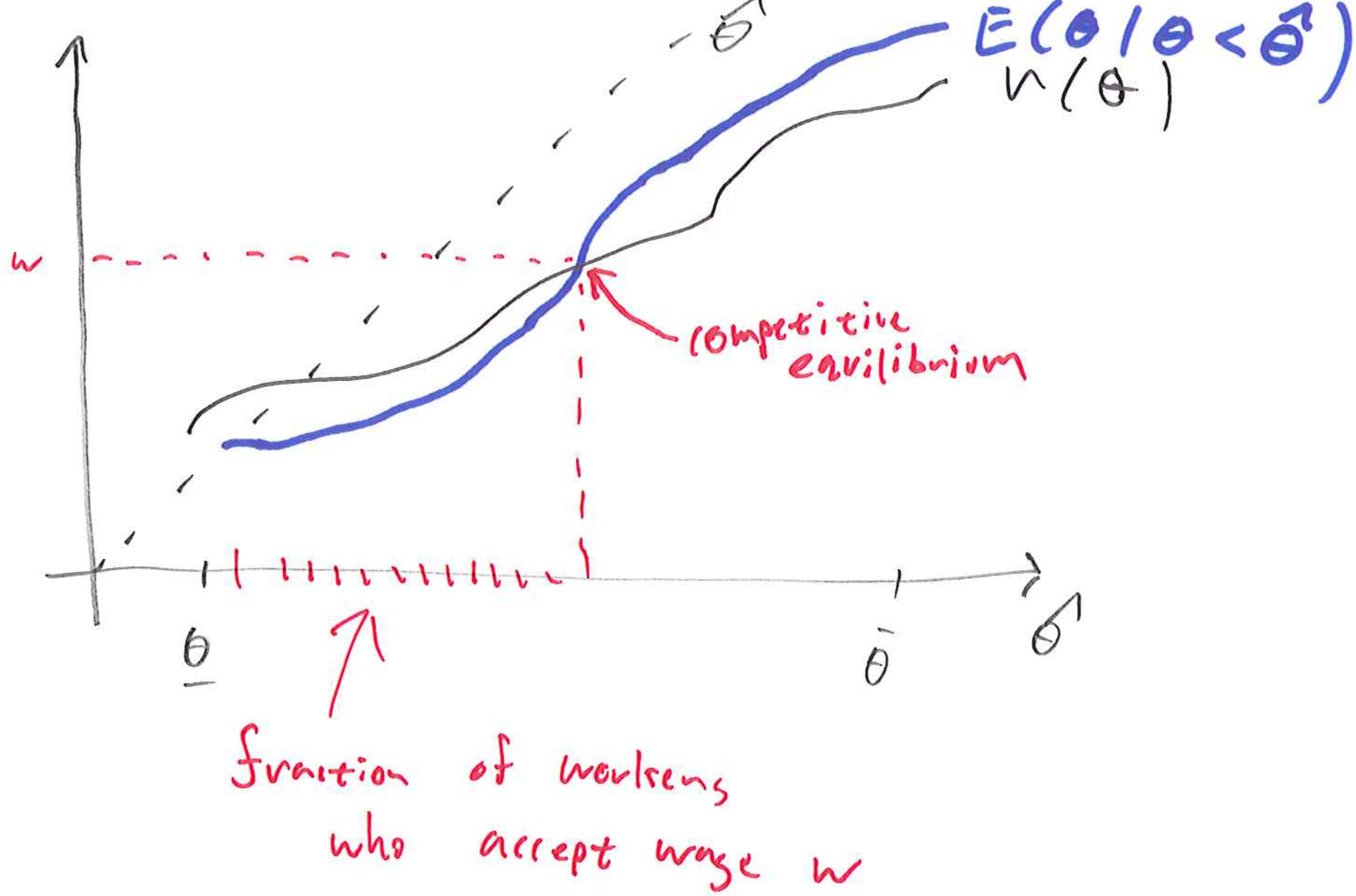
From the answer to part a, as A increases, e^* increases. This is because as the smart worker productivity increases, the wage to being thought smart increases, meaning that normal workers are more likely to want to imitate smart workers. Consequently, smart workers need more education to separate from normal workers.

c. As B increases, does the level of education obtained by smart workers increase or decrease? Explain intuitively why this is the case.

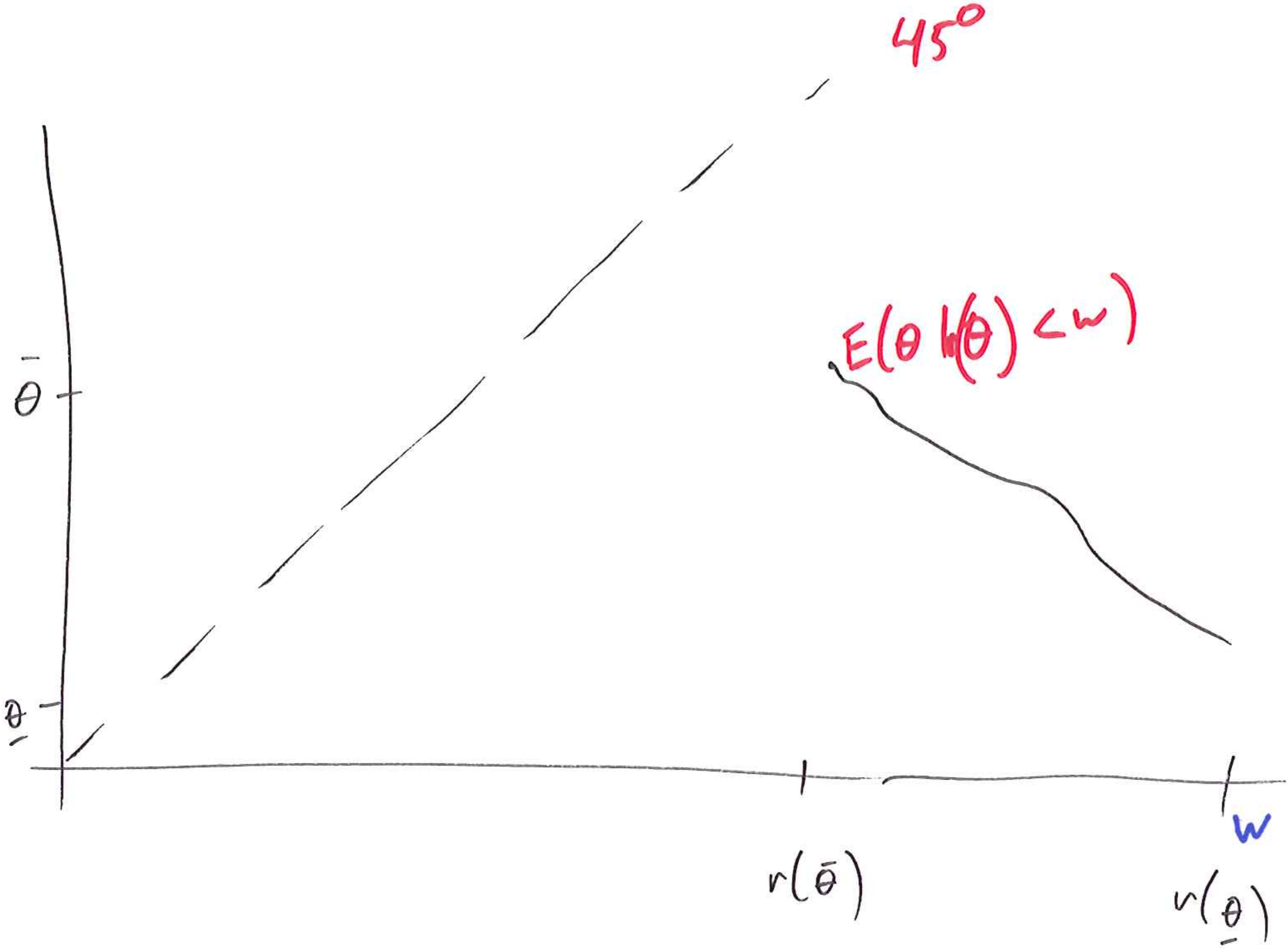
From a., as B increases, e^* decreases. Intuitively, this is because as the disutility of a normal worker associated with a year of education increases, smart types need less education to separate from normal workers, as normal workers are less willing to try to imitate smart workers to get a higher wage.

Problem 6 A house painter has a regular contract to work for a builder. On these jobs, his cost estimates are generally right: sometimes a little high, sometimes a little low, but correct on average. When his regular work is slack, he bids competitively for other jobs. "Those are different," he says. "They almost always end up costing more than I estimate." If we assume that his estimating skills do not differ between the two types of jobs, what can explain the difference?

With either type of job, his estimates will sometimes be too high, and sometimes be too low. With the competitively-bid jobs, however, when his estimates happen to be too high, the customer will simply hire another painter, who by chance had a lower estimate. Therefore, he is only likely to have the lowest bid when he underbids, and so the sample of jobs he does on the weekend is selected. When he estimates costs for his salaried job, there is no selection, so his estimates on average are correct.



$E(\theta \mid \theta < \tilde\theta)$
$r(\theta)$
w
competitive equilibrium
$\underline{\theta}$
$\bar\theta$
$\tilde\theta$
fraction of workers who accept wage w



45°
$E(\theta \mid r(\theta) < w)$
$\bar{\theta}$
$\underline{\theta}$
$r(\bar{\theta})$
$r(\underline{\theta})$
w