

Problem set 2

due 9/30/2009

Problem 1 (War of attrition) (Mas-Collel, problem 9.B.11)

Two firms, A and B, are in a market that is declining in size (e.g. the market for CD's). The game starts in period 0, and the firms can compete in periods 0, 1, 2, ... (i.e. indefinitely) if they so choose. Duopoly profits in period t for firm A are equal to $105 - 10t$, and they are $10.5 - t$ for B. Monopoly profits (those if a firm is the only one left in the market) are $510 - 25t$ for firm A and $51 - 2t$ for firm B.

Suppose that at the start of each period, each firm must decide either to "stay in" or "exit" if it is still active (they do so simultaneously if both active). Once a firm exits, it is out of the market forever and earns zero in each period thereafter. Firms maximize their (undiscounted, for simplicity) sum of profits.

What is this game's subgame perfect equilibrium strategies? What is each firm's profits in the equilibrium/a you identify?

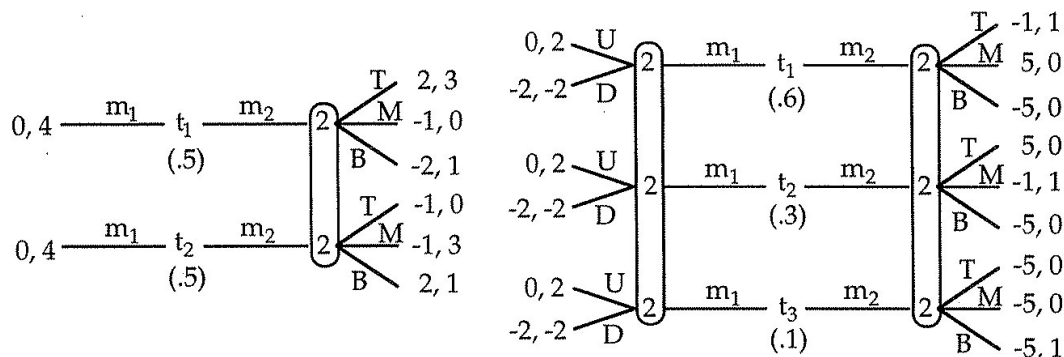
Problem 2 (Nash equilibrium)

Find all Nash equilibria of the following game:

| | | | | |
|----------|----------|----------|----------|----------|
| | | Player 2 | | |
| | | <i>D</i> | <i>E</i> | <i>F</i> |
| Player 1 | <i>A</i> | 0,0 | 5,4 | 4,5 |
| | <i>B</i> | 4,5 | 0,0 | 5,4 |
| | <i>C</i> | 5,4 | 4,5 | 0,0 |

Problem 3 (signalling games)

For each of the two signalling games below, find all perfect Bayesian equilibria, and determine which of them satisfy the intuitive criterion:



Problem 4 (Poker)

Ace-King-Queen Poker is a two-player card game that is played using a deck consisting of three cards: an Ace (the high card), a King (the middle card), and a Queen (the low card). Play proceeds as follows:

1. Each player puts \$1 in a pot in the center of the table.

2. The deck is shuffled, and each player is dealt one card. Each player only sees the card he is dealt.
 3. Player 1 chooses to Raise (R) or Fold (F). A choice of R means that player 1 puts an additional \$1 in the pot. Choosing F means that player 1 ends the game, allowing player 2 to have the money already in the pot.
 4. If player 1 raises, then player 2 chooses to Call (c) or Fold (f). A choice of f means that player 2 ends the game, allowing player 1 to have the money already in the pot. A choice of c means that player 2 also puts an additional \$1 in the pot; in this case, the players reveal their cards and the player with the higher card wins the money in the pot.
- (i) Draw the extensive form of this game.
- (ii) Find all perfect Bayesian equilibria of this game.
- (iii) If you could choose whether to be player 1 or player 2 in this game, which player would you choose to be?
- (iv) Suppose we modify the game as follows: Instead of choosing between Raise and Fold, player 1 chooses between Raise and Laydown (L). A choice of L means that the game ends, the players show their cards, and the player with the higher card wins the pot. Answer parts (ii) and (iii) for this modified game.

In answering this question, refer to player 1's card as A, K, or Q, and refer to player 2's card as a, k, or q.