Problem set 5

due 11/18/2009

Problem 1 (Monopolistic competition with different marginal costs). Suppose that at time t, a representative consumer has preferences over a numeraire good $c_0(t)$ and a bundle of differentiated goods $C(t)$ given by:

$$U = \int_0^\infty (c_0(t) + logC(t))e^{-rt}dt \tag{1}$$

where the bundle of differentiated goods is described by

$$C(t) = \left[\int_0^{n(t)} y(z,t)^\rho dz\right]^{1/\rho} \tag{2}$$

a. Show that the firm producing differentiated good i has demand equal to:

$$y(i,t) = \frac{p(i,t)^{-\sigma}E}{\int_0^{n(t)} p(z,t)^{1-\sigma}dz} \tag{3}$$

where $p(i,t)$ is the price of good i in time t, $\sigma = 1/(1-\rho) > 1$ is the elasticity of substitution between any two differentiated goods, and $n(t)$ is the total number of firms in the differentiated goods sector at time t.

b. Show that if firm i has marginal cost c_i, and given the demand curve in a., firm i sets its price, p_i, such that

$$p_i = \frac{\sigma}{\sigma - 1}c_i \tag{4}$$

c. Finally, show that if there if there are $n_N(t)$ total firms with marginal cost c_N and $n_D(t)$ total firms with marginal cost c_D, firm i earns a profit equal to

$$\pi_i(t) = \frac{c_i^{1-\sigma}}{[c_N^{1-\sigma}n_N(t) + c_D^{1-\sigma}n_D(t)]\sigma} \tag{5}$$

Problem 2 (Adverse selection) A seller has one unit of a good which she may sell to a buyer. the seller has private information about her valuation of the good, v, which is drawn from $[0,1]$ according to the uniform distribution. When the seller's valuation of the good is v, the buyer's valuation is kv, where $k > 1$. *Ex ante*, the buyer does not observe his valuation, however he does have accurate knowledge of the distribution of the seller's valuation. Both players are risk neutral.

a. Suppose that the buyer makes a take-it-or-leave-it offer to the seller. That is, the buyer offers a price at which he is willing to buy, the seller either accepts or rejects, and rejection results in no sale. Describe all subgame perfect equilibria in pure strategies. How does your analysis depend on the value of k?

b. Suppose now that the seller makes a take-it-or-leave-it offer. That is, the seller charges a price, the buyer either accepts or rejects, and rejection results in no sale. Describe all perfect Bayesian equilibria in pure strategies. How does your analysis depend on the value of k?