Online appendices to Ahn T, Sandford J, and Shea P (2023) Lethality and deterrence in affairs of honor. *Rationality and Society*, forthcoming.[1]

A Comparative statics and numerical simulations

This section formally presents the game theoretic model described in section 3 of the main text. We first introduce the functional form assumptions that reflect the verbal descriptions of section 3. We then discuss two comparative statics results that establish the relationship between dueling mortality and other model parameters on outcomes. Our main result is that under any social welfare function that is decreasing in libel and dueling deaths, the optimal mortality in an affair of honor is positive.

There is a population of agents with type uniformly distributed between 0 and 1, who randomly match into pairs. Each pair of agents engages in a public contest, with exactly one winner. The agents in any one match have exogenously specified types θ_1 and θ_2 (both in $[0, 1]$). Without loss of generality, assume that $\theta_1 < \theta_2$. As described in the main text, an agent's type captures his positions on relevant issues, his charisma, his past reputation, and any other aspects that might affect the likelihood that he wins the contest.

While each agent can perfectly observe his opponent's type, public perception of type is subject to distortion by public attacks. Specifically, suppose that each agent can libel his opponent to make him seem more extreme, and less in line with public opinion. Assume that producing amount l of libel costs an agent $c * l^2$, where c is a positive constant. Suppose further that a duel, instigated by either party, decreases some of the effect of this libel. Because duels typically resulted from perceived injuries to one's personal honor, we assume that libel only affects the individual's utility, and not the utility of others with a similar type. Likewise, for a duel to reduce the costs of libel, one had to participate and thus risk death. This eliminates the potential for free-riding where agents benefit from duels that they are not a part of. Let θ_i^p be the perceived type of agent i after being subjected to amount l_j libel:

$$\theta_i^p = \begin{cases} \theta_i - \theta_i \gamma \frac{l_j}{1+l_j} & \text{if no duel takes place} \\ \theta_i - \theta_i (1-\alpha) \gamma \frac{l_j}{1+l_j} & \text{if a duel takes place} \end{cases} \quad (1)$$

where $\gamma \in (0, 1)$ measures the effectiveness of libel, and $\alpha \in (0, 1)$ measures the effectiveness of a duel in reducing libel.

The larger α is, the larger the effect of a duel on perceived type. The parameter α is taken as exogenous by the individuals participating in a duel. In our context, α in the Antebellum South would be larger than in the post-Burr/Hamilton North, for example. We note that it was rare in the Antebellum South for a challenge to be refused. We assume that refusing a challenge was sufficiently costly that any agent would optimally choose to accept.[2]

We assume that each agent's probability of winning the public conflict $\pi_i(\theta_i, \theta_j)$ is increasing in

[1] Available at www.jasandford.com/dueling.pdf.

[2] The refusal of a duel may have had repercussions beyond the current contest, as one's reputation may be marred permanently: "Refusing took nerve, since it was often followed by "posting" in a newspaper or other public place, as today in a gentlemen's club a member may be posted for not paying his bar bills, announcing to the world that X was a cowardly poltroon." (Holland, 2003, pg. 49)

his perceived type and decreasing in his opponent's:[3]

$$\pi_i(\theta_i^p, \theta_j^p) = \frac{1}{2} + \frac{\theta_i^p - \theta_j^p}{2} \tag{2}$$

We finally assume that agents have the following utility function.

$$U_i(\cdot) = \begin{cases} \pi_i - c * l_i^2 & \text{if no duel takes place} \\ \pi_i - dA - c * l_i^2 & \text{if a duel takes place} \end{cases} \tag{3}$$

where $A \geq 0$ is the disutility of death relative to the utility of winning the contest and d is the probability of dying in a duel.[4, 5] The game proceeds in two stages. In stage 1, agents simultaneously choose their levels of libel. In stage 2, agents simultaneously choose whether or not to issue a challenge to duel. In stage 2, agents are fully aware of libel levels from stage 1. Finally, for simplicity assume that an agent who is indifferent between issuing a challenge and not declines to do so.

As discussed in section 3.1, for any pair of types, three varieties of subgame perfect equilibria are possible. First, in an *unconstrained equilibrium*, the potential for dueling may have no effect on the model. This necessarily occurs for sufficiently high values of $d * A$. Effectively outlawing dueling is equivalent to setting $d * A$ to be very high. Second, in a *deterrence equilibrium*, a duel does not occur, but the threat of a duel can deter an agent from choosing the level of libel that he would prefer without dueling. In this case, dueling reduces libel costlessly. Third, in a *dueling equilibrium*, a duel occurs. Because dueling reduces the benefit of libel, a dueling equilibrium also results in reduced effective libel, $(1 - \alpha)l_i$ relative to an outcome without dueling.

A.1 Results

We solve for the subgame perfect equilibria of the 2-stage game described above. Consider the stage 2 decision of each agent, that of whether or not to challenge the other to a duel. Agent i takes libel decions as given and weighs his utility from dueling against that from no duel:

$$\begin{aligned} & U_i^{\text{duel}} \geq U_i^{\text{no duel}} \\ \iff & \theta_i \frac{l_j}{1 + l_j} - \theta_j \frac{l_i}{1 + l_i} \geq \frac{2dA}{\alpha\gamma} \end{aligned} \tag{4}$$

By inspection, the left-hand side of (4) is less than 1 for all values of θ_i and θ_j. This implies two facts. One, given that $\alpha < 1$ and $\gamma < 1$, the condition $d * A > \frac{1}{2}$ is sufficient for a duel being

[3]Equations (1) and (2) imply that, conditional on no duel taking place, the probability of winning the public conflict is represented by a contest success function, as defined by axioms A1-A3 of Skepardas (1996). Allowing for duels, the conflict is not a Skepardas contest, in that a greater amount of libel l_i, by triggering a challenge, may lower agent i's probability of winning the conflict. For a survey of the literature on contests, see Konrad (2009).

[4]Our utility function assumes that agents still receive utility from potentially winning the contest even if they are felled on the field of honor. An alternate approach is to assume that they obtain no such utility by multiplying π_t by $(1 - d)$ in their utility functions. The paper's conclusions are unaffected by this change.

[5]We assume that the probability of dying in a duel, d, is the same for all agents, i.e. that an agent's "skill" does not affect his probability of killing his opponent in a duel. Our reading of the historical record (see in particular Section 2.4) suggests that luck was likely far more important than skill in determining dueling outcomes. The record is, of course, insufficient to conclude that skill was of literally no relevance to the outcome of a duel. Were perceptions of a rival's skill allowed to vary within the model, the threat of a challenge from a more skilled rival would provide a greater deterrent against libeling that agent, all else equal.

suboptimal. Two, the condition $\alpha < 2 * d * A$ is also sufficient for a duel being suboptimal. The former says that if the likelihood of dying in a duel is sufficiently high, no one will choose to duel. The latter says that if the effectiveness of a duel in mitigating libel is sufficiently low, dueling is never preferred. Further, if $\theta_i \frac{l_j}{1+l_j} > \theta_j \frac{l_i}{1+l_i}$, then so long as the quantity dA is sufficiently small, agent i will prefer a duel, whereas agent j will not.

Indeed, equation (4) divides the (θ_1, θ_2) parameter space into three regions. If $\theta_2 < \frac{2dA}{\alpha\gamma}$, there do not exist libel levels that would provoke a challenge. If $\frac{2dA}{\alpha\gamma} \in (\theta_1, \theta_2)$, Agent 1 would never issue a challenge, but Agent 2 would for sufficiently high values of l_1. Finally, if $\frac{2dA}{\alpha\gamma} < \theta_1$, there are values of libel (l_1, l_2) for which Agent 1 issues a challenge (l_2 high relative to l_1), there are libel values for which Agent 2 issues a challenge, and there are libel values for which neither player issues a challenge.

It is apparent that equation (4) holds for at most one agent and only if his opponent's libel is sufficiently large. Let $l_i^{max}(l_j)$ denote the amount of Agent i's libel which would leave Agent j indifferent between issuing a challenge and not. From (4), $l_i^{max}(l_j)$ is defined by:

$$l_i^{max}(l_j) = \begin{cases} \infty & \text{if } \theta_j \leq \frac{2dA}{\alpha\gamma} + \theta_i \frac{l_j}{1+l_j} \\ \frac{\frac{2dA}{\alpha\gamma} + \theta_i \frac{l_j}{1+l_j}}{\theta_j - \frac{2dA}{\alpha\gamma} - \theta_i \frac{l_j}{1+l_j}} & \text{if } \theta_j > \frac{2dA}{\alpha\gamma} + \theta_i \frac{l_j}{1+l_j} \end{cases} \tag{5}$$

Given our assumed tie-breaking rule, the outcome of the second stage will be a duel if and only if $l_1 > l_1^{max}(l_2)$ or $l_2 > l_2^{max}(l_1)$.

Working backwards to stage 1, we describe Agent i's best response to a given l_j. First, if there is no chance of a duel in the second round, Agent i will equate the marginal benefit of libel (distorting j's position) with the marginal cost. In this case, Agent i is unconstrained, and we refer to the resulting libel level as l_1^*. From equation (3), l_i^* is given by:

$$l_i^* : l_i^* + 2\left(l_i^*\right)^2 + \left(l_i^*\right)^3 = \frac{\theta_j \gamma}{4c} \tag{6}$$

An unconstrained best response to a given l_j thus occurs if and only if $l_i^* \leq l_i^{max}(l_j)$ and $l_j \leq l^{max}(l_i^*(l_j))$.

Now suppose that either $l_i^* > l_i^{max}(l_j)$ or $l_j > l^{max}(l_i^*(l_j))$. If at least one of these inequalities hold, it is suboptimal for Agent i to play unconstrained libel level l_i^*, as such a libel level would surely induce a duel, lowering the marginal value of libel. Indeed, conditional on a duel taking place, $i's$ preferred libel level is given by l_i^{**}, where:

$$l_i^{**} : l_i^{**} + 2\left(l_i^{**}\right)^2 + \left(l_i^{**}\right)^3 = \frac{\theta_j(1-\alpha)\gamma}{2c} \tag{7}$$

Suppose first that $l_j > l_j^{max}(l_i^*)$. This means that if Agent i plays his unconstrained libel level l_i^*, Agent j's libel is so great that Agent i will prefer to challenge him. Because $l_j^{max}(l_i)$ is increasing in l_i, it follows that $l_j > l_j^{max}(l_i)$ for any $l_i < l_i^*$ as well. In this case, Agent i recognizes that he will certainly challenge Agent j to a duel in the second stage and plays l_i^{**} and plays l_i^{**} as a best response to l_j.

Now suppose that $l_j < l^{max}(l_i^*(l_j))$, but $l_i^* > l_i^{max}(l_j)$. Here, Agent i's unconstrained libel level is sufficient for j to issue a challenge; that is, $l_i^*(l_j) > l_i^{max}(l_j)$. In this case, Agent i must decide whether to play $l_i^*(l_j) > l_i^{max}(l_j)$, and avoid a duel, or whether to play l_i^{**} and anticipate a duel. If

$l_i^{**} < l_i^*(l_j) > l_i^{max}(l_j)$, his choice is simple; there is no advantage in this case to switching to l_i^{**}, his preferred libel level conditional on a duel, as it will not induce a duel and so is suboptimal. If, however, $l_i^{**} > l_i^*(l_j) > l_i^{max}(l_j)$, then Agent i weighs the utility from dueling, given in equation (8), against his utility from not dueling, given in equation (9):

$$\text{duel: } \pi_i(l_i^{**}, l_j) - dA - c(l_i^{**})^2 \tag{8}$$

$$\text{no duel: } \pi_i(l_i^{max}(l_j), l_j) - c(l_i^{max}(l_j))^2 \tag{9}$$

Equation (10) gives Agent i's best response to a given l_j.

$$BR_i(l_j) = \begin{cases} l_i^* & \text{if} \quad l_i^* \leq l_i^{max}(l_j) \text{ and } l_j \leq l_j^{max}(l_i^*) \\ l_i^{max}(l_j) & \text{if} \quad l_i^* > l_i^{max}(l_j) \text{ and } \\ & \quad \pi_i(l_i^{max}(l_j), l_j) - c(l_i^{max}(l_j))^2 \geq \pi_i(l_i^{**}, l_j) - dA - c(l_i^{**})^2 \\ l_i^{**} & \text{else} \end{cases} \tag{10}$$

In the model's subgame perfect equilibria, agents decide whether or not to duel in stage 2 according to (5), and play mutual best responses in stage 1 according to (10). Our first result is to show that the more moderate Agent 2 is never constrained in equilibrium, rather playing either his unconstrained libel level, or, anticipating a duel in stage 2, l_2^{**}. Lemma 1 establishes this fact.

Lemma 1. *In any subgame perfect equilibrium, Agent 2 plays either l_2^* or l_2^{**}. There is no equilibrium in which Agent 2 is constrained in his libel choice.*

Proof: From equation (6) and the fact that $\theta_1 < \theta_2$, $l_2^* < l_1^*$. Given this, $l_1^{max}(l_2^*) < l_2^{max}(l_1^*)$. Hence, it is not possible for Agent 2 to be constrained without Agent 1 also being constrained. Therefore, Agent 2 is constrained if and only if $l_2^{max}(l_1^{max}(l_2^*)) \leq l_2^*$. However,

$$\begin{aligned} & l_2^{max}(l_1^{max}(l_2^*)) > l_2^* \\ \iff & \frac{\frac{2dA}{\alpha\gamma} + \theta_2 \left(\frac{\frac{2dA}{\alpha\gamma} + \theta_1 \frac{l_2^*}{1+l_2^*}}{\theta_2} \right)}{\theta_1 - \frac{2dA}{\alpha\gamma} - \theta_2 \left(\frac{\frac{2dA}{\alpha\gamma} + \theta_1 \frac{l_2^*}{1+l_2^*}}{\theta_2} \right)} > l_2^* \\ \iff & \frac{4dA}{\alpha\gamma}(1 + l_2^*) > 0 \end{aligned}$$

Because all parameters are assumed to be positive (which also implies $l_2^* > 0$), it must be that $l_2^{max}(l_1^{max}(l_2^*)) > l_2^*$, meaning that if Agent 1 is constrained, it is impossible for Agent 2 to also be constrained. ■

Applying lemma 1 to the best response curve in (10) yields only three possibilities for equilibrium behavior. All of the model's equilibria fall into one of three categories:

1. (unconstrained equilibrium): Agents play (l_1^*, l_2^*) in stage 1, and do not duel in the second stage.

2. (dueling equilibrium): Agents play (l_1^{**}, l_2^{**}) in stage 1, and Agent 2 issues a challenge in the second stage.

3. (deterrence equilibrium): Agents play $(l_1^{max}(l_2^*), l_2^*)$ in stage 1, and do not duel in stage 2. For Agent 1, the marginal benefit of libel exceeds the marginal cost, but he is constrained by the threat of a duel.

Proposition 2 establishes that all three equilibrium types occur for some subset of the parameter space, and characterizes in which portion of the model's parameter space a duel takes place. The results follow from comparing (8) and (9), and from comparing unconstrained libel l_i^* to constrained libel $l_i^{max}(l_j)$.

Proposition 2. *The model's parameter space, over parameters d, α, c, γ, A, θ_1, and θ_2, can be divided into three regions:*

- *Unconstrained region: agents play (l_1^*, l_2^*), and no duel takes place.*
- *Deterrence region: agents play $(l_1^{max}(l_2^*), l_2^*)$, and no duel takes place.*
- *Dueling region: agents play (l_1^{**}, l_2^{**}) and a duel takes place.*

All three regions are non-empty, and the dueling region is characterized by the following:

1. *d, the probability of dying in a duel, is sufficiently low.*
2. *α, the effectiveness of a duel in reducing libel, is neither too low nor too high*
3. *γ, the effectiveness of libel, is sufficiently high*
4. *A, the cost of dying, is sufficiently low*
5. *c, the cost parameter for libel, is sufficiently low*
6. *$\theta_2 - \theta_1$, the difference in moderation between the two agents, is sufficiently large.*

Proof: For an equilibrium duel to occur, it must be that:

$$l_1^* > l_1^{max}(l_2^*) \tag{11}$$
$$\pi_1(l_1^{**}, l_2^{**}) - dA - c(l^{**})^2 > \pi_1(l_1^{max}(l_2^*), l_2^*) - c(l^{max})^2 \tag{12}$$

To show the unconstrained region is non-empty, note that for sufficiently large values of dA, the expected disutility of a duel, $l_1^{max}(l_2^*) = \infty$, meaning that regardless of how large l_1^* is, Agent 2 will never issue a challenge.

To show that the dueling region is nonempty, consider the limiting case of $\theta_2 = 1$ and $\theta_1 = d = A = c = 0$. In this case, $l_1^* = l_1^{**} = \infty$, while $l_1^{max} = 0$ so long as $\alpha > 0$, and $l_2^* = l_2^{**} = 0$. In this case, (11) and (12) above both hold strictly, so long as $\alpha < 1$ and $\gamma > 0$. As both sides of both inequalities above are continuous in all parameters, conditions 1-6 in the statement of the proposition follow.

The examples of section A.1.1 prove the existence of a deterrence equilibrium. ■.

Informally, if α is very low, there is no reason for Agent 2 to issue a challenge ($\lim_{\alpha \to 0} l_1^{max} = \infty$). If α is very close to 1, there is no reason for Agent 1 to libel 2 sufficiently to induce a duel ($\lim_{\alpha \to 1} l_i^{**} = 0$). If c is too large, the benefit to Agent 1 of exposing himself to a challenge (more libel) is too small to cover the cost (chance of annihilation), and condition 2 cannot hold. In the North, where dueling was derided as indulgent nonsense, α was so low that few duels occurred. In the South, where dueling was an acceptable means of conflict resolution, many duels did occur. Duels were also more likely to occur between rivals with very different viewpoints ($\theta_2 - \theta_1$ large).

Section A.3 numerically divides the (θ_1, θ_2) parameter space into unconstrained, deterrence, and dueling regions for a pair of numerical calibrations. First, section A.1.1 below gives three numerical examples that demonstrate how the model is solved.

A.1.1 Numerical examples

First, we show that the following parameterization leads to an unconstrained equilibrium:

$$\theta_1 = \frac{1}{4}, \theta_2 = \frac{3}{4}, \gamma = \frac{1}{2}, \alpha = .9, A = 1, d = .07875, \text{ and } c = \frac{1}{128}$$

First, from (6), we calculate that $l_1^* = 1.6759$ and $l_2^* = 1$. From (5), $l_1^{max}(l_2^*) = 1.72727$. Given $l_1^* < l_1^{max}(l_2^*)$, Agent 1 plays his unconstrained libel level, and no duel takes place in stage 2. The agents' positions are distorted to $\theta_1^p = \frac{3}{16}$ and $\theta_2^p = .51514$, meaning that Agent 1 has a 33.6% chance of winning the contest, versus a 66.4% chance for Agent 2. Agents have utilities of $U_1 = .3142$ and $U_2 = .6560$. Were Agent 2 to issue a challenge, while his probability of winning the contest would increase to 74.14%, his utility would decrease to $U_2 = .6548$, owing to the risk of perishing on the field of honor.

Second, we give parameters that lead to a duel. Consider the following:

$$\theta_1 = \frac{1}{4}, \theta_2 = \frac{3}{4}, \gamma = \frac{3}{4}, \alpha = .9, A = 1, d = .04, \text{ and } c = \frac{1}{128}$$

Here, from (6), we have $l_1^* = 2$ and $l_2^* = 1.2188$. However, $l_1^{max}(l_2^*) = .5177$, meaning that Agent 2 prefers a challenge if Agent 1 plays his unconstrained libel level. If Agent 1 plays $l_1^{max}(l_2^*)$, the agents' perceived types will be $\theta_1^P = .147005$ and $\theta_2^p = .730812$; note in this case that Agent 2 plays a higher libel level than Agent 1, and that Agent 1's type is distorted more than Agent 2's. If constrained, Agent 1's utility is .206, while Agent 2's is .7803.

In the event agents believe a duel will occur in the second stage, Agent 1 optimally plays $l_1^{**} = .6562$, while Agent 2 plays $l_2^{**} = .3361$. In this case, perceived types become $\theta_1^p = .2452$ and $\theta_2^p = .7277$, and Agent 1's utility increases to .2154. Agent 2's utility becomes .7003. Consequently, there is no constrained equilibrium in this case, while there is a dueling equilibrium in which Agent 1 trades off greater relative libel for a small chance of death.

Finally, suppose in the previous example we increase the probability of death in a duel, but leave all other parameters the same:

$$\theta_1 = \frac{1}{4}, \theta_2 = \frac{3}{4}, \gamma = \frac{3}{4}, \alpha = .9, A = 1, d = .1, \text{ and } c = \frac{1}{128}$$

In this case, l_1^* and l_2^* are unchanged from the previous example, but $l_1^{max}(l_2^*)$ increases to 1.3706, as Agent 2 becomes more hesitant to issue a challenge when there is a greater probability of death. In this case, if Agent 1 is constrained, perceived types are $\theta_1^p = .147$ and $\theta_2^p = .7175$, and utilities are $U_1 = .200$ and $U_2 = .774$. Again, Agent 1 has his position distorted relatively more than Agent 2.

If agents anticipate a duel in the second stage, libel levels continue to be $l_1^{**} = .6562$ and $l_2^{**} = .336$. In this case, perceived types become $\theta_1^p = .245$ and $\theta_2^p = .728$, while utility is $U_1 = .155$ and $U_2 .640$. Thus, despite Agent 1's stronger relative position in the event of a duel, the threat of death is too high, and Agent 1 prefers to play his constrained libel level in stage 1, avoiding a duel. Notably, total utility is greater in the equilibrium of the previous example, with a lower probability of death from dueling.

A.2 Optimal mortality

We now focus on one parameter in particular. The probability of dying in a duel, $d \in [0, 1]$, is an important determinant both of whether or not duels take place, and of the welfare consequences of the institution of dueling. Moreover, d is plausibly manipulable by policy and social norms; the use of outdated dueling pistols presumably lowered d relative to more modern weapons, and we could view effectively outlawing dueling as raising d. We argue that, under the assumption that social welfare is decreasing in both the amount of libel and in dueling deaths, the socially optimal mortality d^* is both strictly greater than zero and strictly less than 1. Duels must be dangerous enough so that they do not occur too frequently, but not so dangerous that the threat of a duel is not a credible curb to ungentlemanly behavior.

Proposition 3 shows that we can partition the set $[0, 1]$ into three possibly empty regions, $[0, d_1)$, $[d_1, d_2)$, and $[d_2, 1]$, with $0 < d_1 < d_2 < 1$. For $d \in [0, d_1)$, a duel takes place. For $d \in [d_1, d_2)$, Agent 1 is deterred from libelling Agent 2 enough to provoke a duel. For $d \in [d_2, 1]$, both agents are unconstrained.

Proposition 3. *Equilibrium behavior is described by cutoffs $0 < d_1 < d_2 < 1$ as follows:*

1. *For $d \in [0, d_1)$, a duel occurs.*
2. *For $d \in [d_1, d_2)$, a deterrence equilibrium occurs*
3. *For $d \in [d_2, 1]$, an unconstrained equilibrium occurs.*

Proof: The cutoff d_2 is set such that $l_1^* = l_1^{max}$. As d approaches 1, the condition $A > \frac{1}{2}$ ensures that $l_1^{max} > l_1^*$, meaning that both agents are unconstrained, and so $d_2 < 1$.

For $d = d_2$, $l_1^* = l_1^{max}$. From (6) and (7), $l_1^{**} < l_1^*$, and so the fact that l_1^{max} is a continuous function of d implies that there exists $\epsilon > 0$ s.t. Agent 1 strictly prefers to play l_1^{max} and avoid a duel for $d \in (d_2 - \epsilon, d_2)$, and hence $d_1 < d_2$.

For $d < d_2$, the utility from a duel (8) is decreasing in d, while the utility from no (9) is increasing in d, implying that the partition $[0, d_1)$, $[d_1, d_2)$, $[d_2, 1]$ is valid. Finally, from (8) and (9), as $d \to 0$, Agent 1 prefers to duel to not, by virtue of $l_1^* *$ solving his first order condition (7). ■

The logic of proposition (3) is simple. The cutoff d_2 is set where $l_1^* = l_1^{max}$, or the amount of libel that leaves Agent 2 indifferent between a duel and no duel. Clearly, l_1^{max} is increasing in d, as an increase in d makes a duel riskier and thus causes Agent 2 to tolerate more libel before issuing a call to the field of honor, while l_1^*, representing Agent 1's preferred libel level absent a duel, is invariant in d. Therefore, for $d > d_2$, Agent 1 is unconstrained, and no duel takes place. For $d < d_2$, Agent 1 must choose between playing l_1^{max} and l_1^{**}, but for d in the neighborhood of d_2, Agent 1 will strictly prefer to not induce a duel, and so $d_1 < d_2$. For values of d approaching 1, Agent 2 would never issue a challenge and accept near-certain death, so $d_2 < 1$. Finally, it is clear that as $d \to 0$, dueling becomes a dominant strategy for Agent 2 for any positive level of libel; he can reduce the effectiveness of Agent 1's libel at no real cost to himself, and so $d_1 > 0$.

What are the implications of proposition 3 for social welfare? While views vary on the social (dis)utility of dueling and libel, we assume a social welfare function which is decreasing in both libel and dueling deaths. First, libel distorts the political process, the media, and commerce, and we feel it is reasonable that, all else equal, society is worse off the more libel there is. Second, even those most in favor of dueling acknowledged that, all else equal, dueling deaths were unfortunate, and every effort was made to forestall duels with a settlement acceptable to both sides.[6] Hence, it is worth asking the following: under a social welfare function which is decreasing in both the amount of libel and the number of duels, what is the optimal mortality rate of a duel? This was a question of central importance that society governed, both in that the deadliness of dueling weapons was influenced by agreed-upon customs, and in that the vigor with which law enforcement agencies prosecuted anti-dueling laws greatly affected the cost of dueling. Indeed, we might think of a policy that effectively outlawed dueling as setting the mortality from a duel to $d = 1$, and a regime in which dueling pistols were used instead of rifled percussion cap weapons as artificially lowering the parameter d.

Agent 1's equilibrium libel moves non-monotonically in d, first decreasing, then increasing, while Agent 2's libel increases monotonically. Specifically, Agent 1 plays l_1^{**}, l_1^{max}, and l_1^* in the $[0, d_1)$, $[d_1, d_2)$, and $[d_2, 1]$ regions, respectively. As $l_1^* > l^{**} > l_1^{max}|_{d=d_1}$, Agent 1's libel level is non-monotonic in d, first decreasing (at d_1), then increasing (between d_1 and d_2), then constant (in $[d_2, 1]$). Agent 2 plays l_2^{**} for $d < d_1$ and l_2^* for $d > d_1$, and so Agent 2's libel is increasing in d (jumping up at d_1 and constant otherwise). Total libel, $l_1 + l_2$, may increase or decrease at d_1, depending on the amount of libel levied by Agent 2, though it will decrease for any type pair with sufficiently large $\theta_2 - \theta_1$.

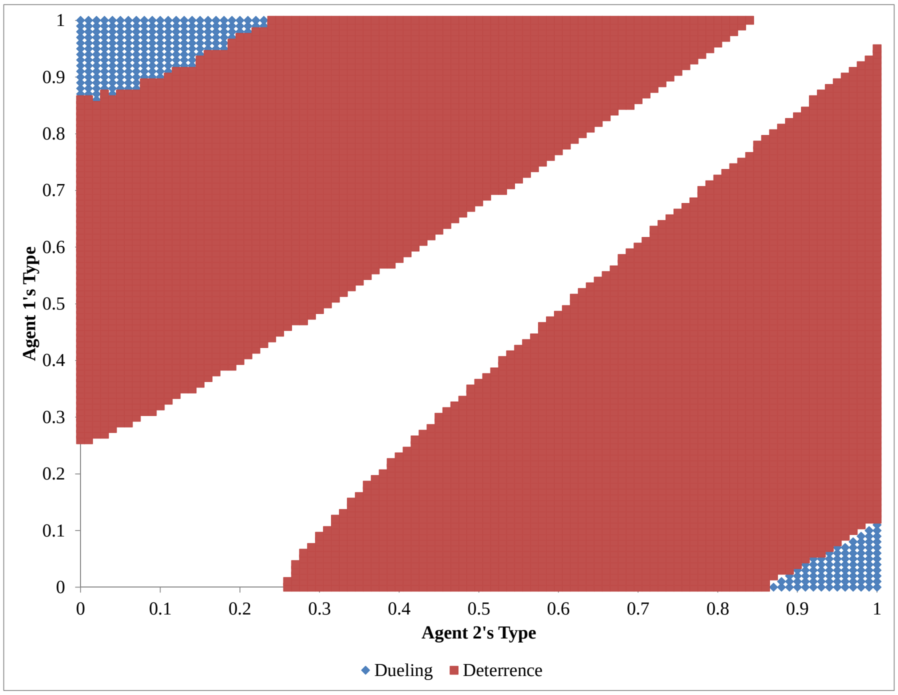
Further, it follows from proposition 3 that dueling deaths are minimized (at 0) for any $d \geq d^1(0, 1)$. Given that total libel is increasing for $d \geq d_1$, it is direct that $d = d_1$ dominates all $d > d_1$ (dueling fatality rates in this range induce the same number of dueling deaths, 0, but produce more libel). In the event that $l_1 + l_2$ for $d < d_1$ is less than at $d = d_1$, it is also unambiguous that $d = d_1$ dominates all $d < d_1$. Finally, in the event that total libel is lower for $d < d_1$, whether d_1 or a lower fatality rate is optimal depends on how society weights the occasional duel or fatality against a lower libel level. If dueling deaths lower social welfare greatly more than libel does, it is unambiguous that $d^* = d_1$. The optimal mortality in an affair of honor is just high enough to deter agents from both higher levels of libel and from actually challenging each other. A higher fatality rate would have less of a deterrent effect, as libeling agents would be less likely to be called to the field of honor, while a lower fatality rate would only encourage needless bloodshed.

Of course, d_1 is different for different pairs of agents; the greater $\theta_2 - \theta_1$, the lower d_1 is. However, since proposition 3 holds for all (θ_1, θ_2) pairs, there necessarily exists some d_1^* for which every possible pair of agents is at least marginally deterred from dueling. Corollary 4 summarizes.

Corollary 4. *For any social welfare function, the optimal mortality from an affair of honor is* $d^* \in (0, 1)$.

[6] US Congressman William Graves, upon killing his colleague Congressman Jonathan Ciley in a duel, described visions of Ciley's ghost, with a bullet wound to his forehead, visiting him at night, and demanded of his wife that they sleep with the lights on to prevent the apparition's visits (New York Times, March 5, 1877).

Figure A.1: Equilibrium by type for high α (South)



Corollary 4 sheds light on why, even at the height of dueling's popularity in the Antebellum South, ineffective and inaccurate dueling pistols were used to settle disputes, as opposed to more modern and deadly weapons. The efficiency of the institution depended on just such a choice.

Finally, we note that the more dueling was accepted by a society, the greater the benefit to be realized from it. Here, we model the acceptance of dueling via the parameter α, with higher α implying a greater acceptance of dueling and it therefore being a better way to clear one's name. Proposition 5 tells us that the more accepted dueling was, the more benefits to be reaped from the institution. The North, where dueling was widely derided, failed to achieve the same benefits from dueling as did the South, where it was widely accepted.

Proposition 5. *If $\gamma > 2dA$, then an increase in α lowers the total amount of libel.*

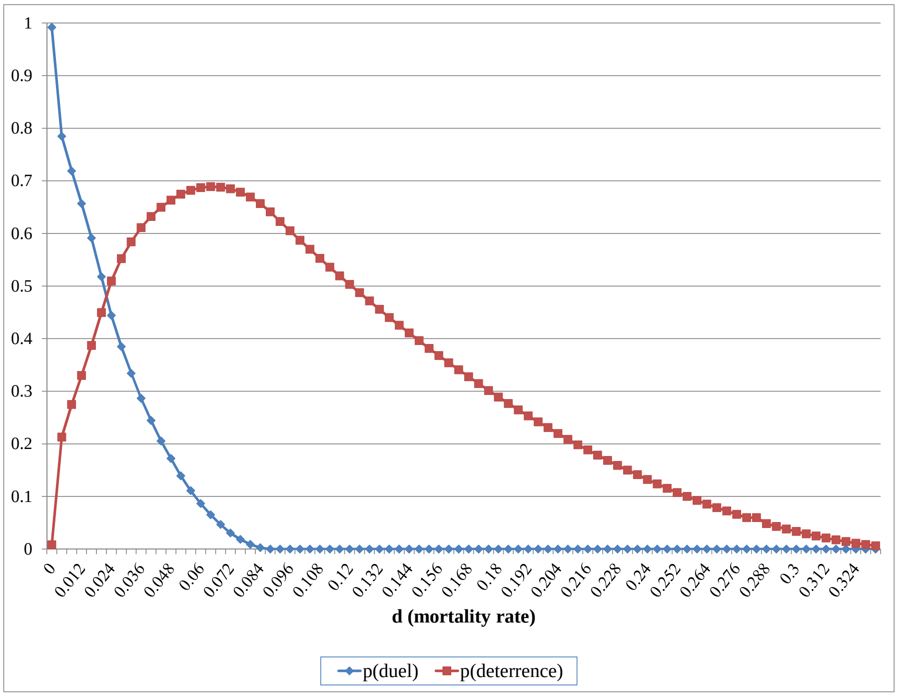
Proof: An increase in α decreases l_1^{max} and l_i^{**}, while not affecting l_i^*. Therefore, for any θ_1, θ_2 pair in either a deterrence or dueling equilibrium, the total libel $l_1 + l_2$ decreases. So long as $\gamma > 2dA$, a positive measure of θ_2, θ_1 pairs are in a deterrence equilibrium. ■

A.3 Numerical simulation

We now simulate the model over a lattice where both θ_1 and θ_2 range between zero and one. We use the following calibration: $\alpha = \frac{1}{2}$, $\gamma = 1$, $A = \frac{1}{2}$, $c = 0.01$, and $d = 0.072$, approximately equal to the estimated mortality rate of $\frac{1}{14}$ discussed in the introduction. Figure A.1 reports the results.

Duels occur when the type difference between agents is large. For these parameterizations, the more extreme agent has a large incentive to libel his opponent. The moderate agent reduces the effects

Figure A.2: Probability of Dueling and Deterrence Equilibria



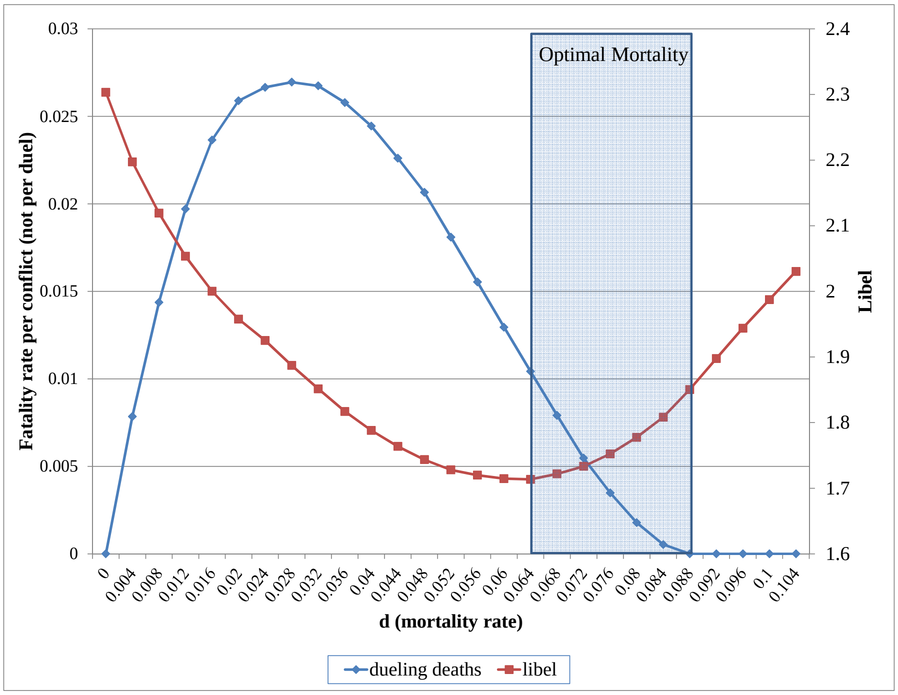
of this libel by meeting his opponent on the field of honor. A deterrence equilibrium exists when the difference in types is intermediate. In this case, the more extreme agent reduces his libel in order to avoid prompting a challenge. When agents have similar types, they choose similar amounts of libel. Neither agent thus has an incentive to issue a challenge and the field of honor lies vacant.

We now consider the effect of different mortality rates. We vary d between 0.001, and 0.35, the latter value being sufficiently high so that the model behaves the same as if dueling were outlawed, for $\alpha = \frac{1}{2}$. Figure A.2 plots the probability of dueling and deterrence equilibria for different mortality rates while figure A.3 shows the average level of libel and the fatality rate per contest (not per duel).

Suppose that social welfare is decreasing in libel and non-increasing in deaths from duels. Figure A.3 provides several insights on the optimal mortality rate. First, the mortality rate that minimizes libelis 6.4%. Second, for any mortality rate less than 6.4%, it is always possible to choose a mortality above 6.4% which results in less libel and fewer dueling deaths. For this calibration, any mortality rate below 6.4% is therefore indefensible. Third, if a society is unwilling to accept any deaths in dueling, then a mortality rate about 8.8% will deter libel without resulting in any actual duels. Fourth, the only effect of increasing the mortality rate above the minimum rate that results in no duels is more libel. Outlawing dueling (equivalent to setting $d \geq 35\%$) thus maximizes libel, and absent an alternative mechanism to deter libel may be suboptimal.

Collectively, these results imply an optimal mortality rate between approximately 6.4% and 8.8% (the shaded region in figure A.3). Within this range, a society faces a tradeoff between more dueling deaths and more libel. It is not the case that making dueling safer will always reduce dueling deaths. Lowering the mortality rate from 5% to 2%, for example, results in more libel while increasing the amount of blood spilt on the field of honor.

Figure A.3: Average Libel and Deaths per Contest



Appendix references

[1] Konrad K (2009) *Strategy and Dynamic Contests*, Oxford University Press.

[2] Skepardas S (1996) Contest success functions. *Economic Theory* 7: 283-29.

B Selected historical facts

List of verified U.S. Senators and other prominent politicians who participated in at least one duel:

Henry Clay, Humphrey Marshall, David C. Broderick, Armistead T. Mason, Andrew Jackson, George A. Waggaman, James Shields, John Randolph, William H. Crawford, John Rowan, George M. Bibb, Thomas H. Benson, James D. Westcott, David Barton, James Gunn, James Jackson, Josiah Johnson, Thomas Clingman, John Fremont, Sam Houston, John Crittenden, Pierce Butler, Thomas Metcalfe, John Adair, Benjamin Gratz Brown, Henry Geyer, Henry Foote, Louis Wigfall, Alexander Buckner, Lewis Linn, Garrett Davis, Jonathan Dayton, George McDuffie, William Gwin, John Breckenridge, James Farley, George Wallace Jones, Harrison Riddleberger, James Hammond, Dewitt Clinton, Edward Lloyd, Robert Wright, Thomas Rusk, George Campbell, Jefferson Davis, William R. King, Gabriel Moore, Clement C. Clay, William C. C. Claiborne, Jeremiah Clemens, Ambrose Sevier, Solon Borland, Aaron Burr, Judah Benjamin, and Franklin Pierce. Senators Pierce, Bibb, Johnson, Crittenden, Adair, and Davis acted as seconds in duels, but may not have ever participated as principals. Senator Linn participated in a friend's duel as a surgeon. Senators Metcalfe, Davis, Dayton, Hammond, Rusk, King, and Benjamin issued calls to the field of honor, but were declined or otherwise unable to come to acceptable terms. Sen. Barton is not known to have been personally involved in a duel, but his brother Joshua was killed in one defending charges the senator had made in a newspaper against a rival. The other 41 acted as principals on the field of honor.

Some descriptions of outcomes of duels:

- Duels claimed the lives of three US senators (one sitting), one signer of the Declaration of Independence, one standing congressman, and naval war hero Stephen Decatur.

- Armistead T. Mason of Virginia was killed by his brother-in-law on 2/6/1819. George A. Waggaman of Louisiana was killed on 3/31/1843. David C. Broderick of California was shot on 9/13/1859 by David S. Terry, a chief justice of the California supreme court, who resigned "to free himself from possible criticism" which might arise upon his shooting Sen. Broderick.

- Button Gwinnett, a signed of the Declaration of Independence, died 5/16/1777 at the hands of Lachlan Macintosh, a brigadier general in the Continental Army.

- George Trotter, while serving as editor of the Kentucky Gazette, objected both to Charles Wickliffe's pro-slavery stance and his having murdered the previous editor. Wickliffe invited Trotter to an interview and was immediately accepted on the condition that the duel be fought at the atypical distance of only 8 feet. Wickliffe was soon dead.

- Hamilton and Burr were old political enemies. Decatur was shot by a former subordinate officer who disagreed with Decatur's assessment of him during a court martial. Andrew Jackson's one fatal duel arose out of a dispute over the propriety of a horse wager, with his eventual victim being goaded into stepping up his dispute with Jackson by one of Jackson's political opponents.

- Abraham Lincoln's near-duel with James Shields arose out of a public dispute over tax policy.

- When Jackson announced his candidacy for the presidency, a political opponent published a pamphlet entitled "The Indiscretions of Andrew Jackson" which claimed Jackson was involved in 14 duels between the ages of 13 and 60 (Seitz, pg. 123). Only one is known to have resulted in a fatality; Jackson killed Charles Dickinson on 5/30/1806. Dickinson had himself killed 26 people in Duels (ibid.).

- President Andrew Jackson fought over a dozen duels without apparent cost to his political career. Among the duels related to his wife Rachael's alleged bigamy was a challenge from Jackson to Tennessee Governor John Sevier in 1803. A letter to Sevier in which he insists that their duel be fought in Knoxville indicates Jackson's motivation behind his challenge. He wrote to Sevier: "In the town of Knoxville did you take the name of a lady into your polluted lips. In the town of Knoxville, when you were armed with a cutlass and I with a cane. And now, sir, in the town of Knoxville you shall atone for it or I will publish you as a coward and a poltroon." (The Papers of Andrew Jackson, Vol. I, 1770-1803. eds. Smith, S. and H. Owsley). Jackson's reference to atonement suggests that he made his challenge to counteract a specific slander and not out of a general sense of honor. Furthermore, were he motivated by a general sense of vengeance, the choice of location would not have concerned him.

- To emphasize the extent to which something other than revenge drove duelers, consider Secretary of State Henry Clay's dispute with Senator John Randolph of Virginia over whether President Adams had the authority to unilaterally appoint ambassadors to a meeting of Latin American states led by Simon Bolivar. After trading insults, Clay invited Randolph to a meeting. Clay punctured Randolph's coat with his shot, after which Randolph fired into the air. They met halfway and jocosely shook hands, Randolph saying "you owe me a new coat, Mr. Clay!" to which Clay responded "I'm glad the debt is no greater! I trust in God, my dear sir, you are untouched; after what has occurred, I would not have harmed you for a thousand worlds!"

Contemporary newspaper articles describing duels:

- From the Raleigh Register, and North Carolina Weekly Advertiser, 3/29/1810: "On Saturday last a duel was fought on the Louisiana side of the Mississippi, opposite Natchez... The ground work of the meeting was laid at New Orleans as far back as last spring, when the parties disputed on the subject of Gen. Wilkinson."

- From the Charleston Mercury, 3/23/1857: "A duel took place at the "Oaks" (near New Orleans) on the 12th inst., between Mr. J. W. McDonald, editor of the Natchez Free Trader, and Capt. J.K. Duncan — pistols being the weapon, and the distance twelve paces. Shots were exchanged without doing any damage. After the first fire, the challenge was withdrawn by Mr. McDonald."

- From the New Orleans Bee, 12/3/1860: "Messrs. Eugene Cuvellier and L.A. Raymond had a hostile meeting at four o'clock on Saturday. The weapons used were sharpened foils, and at the first pass Mr. Cuvellier received a slight wound in the left breast, while his own weapon passed through the shirt of Mr. Raymond. The affair was stopped by the seconds, who arranged an explanation and reconciliation between the two parties. Both of the gentlemen conducted themselves with perfect coolness and bravery.

- From the Newbern Sentinel, 12/18/1819: "On the 14th ult. Captain Albert G. Tomlinson and Mr. David Jeffreys, both of Person county, passed over into Va. to settle an affair of honor. They fought with pistols, and the distance, at which they shot each other, was quite respectful. We are happy to state that no injury was done either of the parties. After an exchange of shot the affair was adjusted to their mutual satisfaction."

- From the Raleigh Register and North Carolina Gazette, 3/4/1848: "Affair of Honor.— The N.O. Picayune of the 20th inst., announces, as follows, the adjustment of a personal difficulty which had excited very painful interest in New Orleans. The parties are understood to have been the Hon. S. S. Prentiss, and Mr. Irving of Kentucky. We do not remember, to have witnessed the same degree of satisfaction of any difficulty of a personal character, as was exhibited yesterday when it was understood that the quarrel... had been satisfactorily arranged. The public seemed to be satisfied, from the character of the parties engaged in the affair, that the grounds of adjustment were sufficient, and all curiosity as to the precise terms of the settlement, were buried beneath a universal feelings of delight at the assurance that the parties themselves and their friends were entirely content with the result.